

# Holographic helical superconductors

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## Abstract

We show that  $D = 5$  electrically charged  $AdS$  black branes can have instabilities associated with spatially modulated  $p$ -wave superconductors with helical structure. We show that the instabilities are present within  $N = 4$   $SU(2) \times U(1)$  gauged supergravity and hence within  $D = 10$  and  $D = 11$  supergravity.

# 1 Introduction

The AdS/CFT correspondence is a powerful framework for studying the properties of strongly coupled quantum critical points, with potentially important applications to condensed matter systems such as strongly correlated electrons or cold atoms. One focus of activity in such AdS/CMT studies has been the construction of electrically charged *AdS* black holes, describing CFTs at finite temperature and charge density, that are holographically dual to superconducting states, or more precisely to superfluid states.

Black holes corresponding to *s*-, *p*- and *d*-wave superconductors, whose order parameters have angular momentum  $l = 0, 1$  and  $2$ , respectively, have all been constructed. In the *s*-wave superconducting black holes, the bulk charged fields dual to the order parameter for the superconductivity, are scalar fields. They have been constructed in phenomenological theories of gravity in [1–3] and then in  $D = 10, 11$  supergravity in [4–7]. *p*-wave superconducting black holes have been constructed using either charged vector fields in the bulk [8–10] or, alternatively, charged two-forms [11], but not yet within  $D = 10/11$  supergravity<sup>1</sup>. *p*-wave superconductivity is seen in some heavy fermion systems, such as  $UPt_3$ , in  $Sr_2RuO_4$  (e.g. see [15]) and in some organic materials, such as the Bechgaard-salt  $(TMTSF)_2PF_6$ . *p*-wave superfluids are seen in  $He_3$  and have also been recently observed in Fermi-gases (e.g. see [16]). Holographic *d*-wave superconducting black holes have been constructed in [17] using charged massive spin two-fields. The embedding of the *d*-wave superconductors into  $D = 10, 11$  supergravity is problematic because the consistency of interacting massive spin two-fields requires an infinite number of bulk fields. The superconductivity in the high  $T_c$  cuprates is well known to be of *d*-wave type.

All of these black hole solutions describe spatially homogeneous superconducting states. However, it has long been known that it is possible to have superconducting states that are spatially inhomogeneous. Indeed in the Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) phase, a Cooper pair consisting of two fermions with different Fermi momenta condenses leading to an order parameter with non-vanishing total momentum [18, 19]. There are several systems in which it has been argued that the FFLO state is present but, it seems, not without some controversy.

A second focus of activity in AdS/CMT studies has been the construction of black holes that are dual to (non-superconducting) spatially modulated phases, which are also widely seen in condensed matter systems. It has been shown that electrically

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<sup>1</sup>*p*-wave superconductors have been discussed in the context of *D*-brane probes in [12–14].

charged black holes in  $D = 5$  can have instabilities corresponding to spatially modulated current density waves in  $D = 5$  in [20–22] (for earlier related work see [23]). Similar instabilities are also present in  $D = 4$  [24] and in this case they are associated with “striped” black brane solutions which are dual to phases with both current density waves and charged density waves, with the wave number for the latter being twice that of the former. Examples of these instabilities were shown to exist in  $D=10,11$  supergravity in [24], while analogous instabilities were observed in a probe-brane setting in [25]. Most recently, it has been shown that magnetically charged black branes can also have spatially modulated instabilities [26].

The purpose of this paper is to combine these two lines of development and show that spatially modulated superconducting states are possible within the context of AdS/CFT and moreover that they exist within string/M-theory. The examples we discuss will be in the context of  $D = 5$  theories of gravity and correspond to spatially modulated  $p$ -wave superconductors with a helical structure in the dual  $d = 4$  CFT. We will provide helical generalisations of both holographic  $p_x$ -wave order [9] and  $(p_x + ip_y)$ -wave order [8]. We will show that helical superconductors are possible using two distinct mechanisms in  $D = 5$ .

We first study each mechanism in simplified phenomenological models of gravity in  $D = 5$ . In the first model, the bulk fields consist of a metric, a  $U(1)$  gauge field and the order parameter, spontaneously breaking the dual global  $U(1)$  symmetry, is provided by a charged two-form satisfying a self-duality equation, similar to [11]. In the second model they consist of a metric, a  $U(1)$  gauge-field and  $SU(2)$  gauge-fields as in [27]. For this model, generically the black branes are charged with respect to  $U(1) \times U(1) \subset U(1) \times SU(2)$  and the order parameter is provided by the charged  $SU(2)$  gauge-fields, breaking  $U(1) \times U(1) \rightarrow U(1)$ . For one particular case, the background is just charged with respect to the  $U(1)$  factor and hence preserves the full  $U(1) \times SU(2)$  global symmetry. For this case the instability preserves the  $U(1)$  symmetry but breaks the  $SU(2)$  symmetry, reminiscent of what is seen in spiral spin density waves.

We will also show that helical superconducting black holes appear in Romans’  $N = 4^+$   $SU(2) \times U(1)$  gauged supergravity theory [28]. This is significant because Romans’ theory arises as a consistent Kaluza-Klein (KK) truncation of type IIB supergravity on an  $S^5$  [29], thus capturing a sector of  $N = 4$   $d = 4$  SYM, and also of  $D = 11$  supergravity on the general class of  $M_6$  [30] (see also [31]) associated with  $N = 2$   $d = 4$  SCFTs [32]. Thus, by showing that the superconducting black holes are present in Romans’ theory we will have demonstrated the existence of

helical superconductors in  $D = 10, 11$  supergravity. We will calculate the critical temperature at which the superconductivity involving the  $SU(2)$  gauge-fields appears in Romans' theory. Our numerical techniques are not stable enough to determine whether or not the superconductivity involving the two-form also occurs in Roman's theory; if it does it will be at a much lower temperature. On the other hand, we will show that Romans' theory also has a neutral instability of Gubser-Mitra type [33, 34], which appears at a higher temperature. This model thus has a rich structure and it will be interesting to study it in more detail, generalising the analysis of [35].

Our strategy is to analyse linearised perturbations about electrically charged  $AdS$  black branes in  $D = 5$  with no charged hair. We will only consider black branes, such as the AdS-RN black branes, which at zero temperature approach an  $AdS_2 \times \mathbb{R}^2$  solution in the near horizon limit. We first look for spatially modulated instabilities by studying perturbations that violate the  $AdS_2$  BF bound. This provides a sufficient (but not necessary) criteria for instabilities of the full black brane solutions. For some representative examples, we then construct normalisable zero modes about the full black brane solutions and determine the critical temperatures at which the spatially modulated instabilities set in.

## 2 The charged two-form model

We consider a  $D = 5$  theory coupling a metric to a gauge field  $A$  and a complex two-form  $C$  with Lagrangian

$$\mathcal{L} = (R + 12) * 1 - \frac{1}{2} * F \wedge F - \frac{1}{2} * C \wedge \bar{C} - \frac{i}{2m} C \wedge \bar{H}, \quad (2.1)$$

where a bar denotes complex conjugation and the field strengths are

$$F = dA, \quad H = dC + i \frac{q}{\sqrt{3}} A \wedge C. \quad (2.2)$$

The equations of motion are given by

$$\begin{aligned} R_{\mu\nu} &= -4g_{\mu\nu} + \frac{1}{2} \left( F_{\mu}{}^{\rho} F_{\nu\rho} - \frac{1}{6} g_{\mu\nu} F_{\rho\sigma} F^{\rho\sigma} \right) + \frac{1}{2} \left( C_{(\mu}{}^{\rho} \bar{C}_{\nu)\rho} - \frac{1}{6} g_{\mu\nu} C_{\rho\sigma} \bar{C}^{\rho\sigma} \right) \\ d * F &= -\frac{q}{2\sqrt{3}m} C \wedge C \\ *H &= imC \end{aligned} \quad (2.3)$$

When  $q = m = 1$ , the black hole backgrounds and the *linearised* perturbations that we consider for this model, are also relevant for Romans' theory of  $N = 4^+ SU(2) \times U(1)$

gauged supergravity, as we discuss in section 4. In [11] spatially homogeneous  $p$ -wave superconductivity was investigated with  $m = 1$  and variable  $q$  in a similar context. Here we investigate spatially modulated  $p$ -wave superconductivity for general  $m, q$ .

The equations of motion (2.3) admit a unit radius  $AdS_5$  solution with vanishing matter fields, that is dual to the vacuum state of a  $d = 4$  CFT with a global  $U(1)$  symmetry. The gauge-field  $A$  is dual to the conserved current associated with the global  $U(1)$  symmetry and has dimension  $\Delta = 3$ . The equation of motion for the two-form  $C$  describes three complex massive propagating degrees of freedom corresponding to self-dual tensor operators<sup>2</sup> of dimension  $\Delta_{\pm} = 2 \pm m$  in the dual CFT.

The equations of motion also admit the electrically charged AdS-RN black brane solution

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 (dx_1^2 + dx_2^2 + dx_3^2), \quad A = a(r) dt, \quad (2.4)$$

where

$$f = r^2 - \frac{r_+^4}{r^2} + \frac{\mu^2}{3} \left( \frac{r_+^4}{r^4} - \frac{r_+^2}{r^2} \right), \quad a(r) = \mu \left( 1 - \frac{r_+^2}{r^2} \right). \quad (2.5)$$

Here  $\mu$  is a chemical potential for the global  $U(1)$  symmetry in the dual CFT. The (outer) event horizon is located at  $r = r_+$  and the Hawking temperature is given by  $T = (6r_+^2 - \mu^2)/6\pi r_+$ . At zero temperature, when  $\sqrt{6}r_+ = \mu$ , the near horizon limit is the  $AdS_2 \times \mathbb{R}^3$  solution

$$ds^2 = L^2 \left( -\rho^2 dt^2 + \frac{d\rho^2}{\rho^2} \right) + dx_1^2 + dx_2^2 + dx_3^2, \quad A = \frac{1}{\sqrt{6}} \rho dt, \quad (2.6)$$

with radius squared given by  $L^2 = 1/12$  and we have rescaled the coordinates  $t \rightarrow \frac{1}{12}t$ ,  $x_i \rightarrow (\sqrt{6}/\mu) x_i$ .

## 2.1 Instabilities for $AdS_2 \times \mathbb{R}^3$

Consider the perturbation of the two-form around the  $AdS_2 \times \mathbb{R}^3$  solution (2.6) given by

$$\delta C = dr \wedge (u_1 dx_1 - v_1 dx_2) + i dt \wedge (u_2 dx_1 - v_2 dx_2) + dx_3 \wedge (u_3 dx_2 + v_3 dx_1), \quad (2.7)$$

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<sup>2</sup>When Romans' theory is considered as a consistent truncation of type IIB supergravity, the  $\Delta = 3$  operator dual to the two form  $C$  has been identified in  $N = 4$   $d = 4$  SYM as being  $Tr \Phi F_{\mu\nu}^+$ , where  $\Phi$  is a complex scalar and  $F^+$  is the self-dual part of the YM field strength [11].

where  $u_i, v_i$  are complex functions of  $t, \rho$  and  $x_3$ . It is straightforward to show that this decouples from other perturbations at linearised order. We will consider the following two types of spatially modulated modes. In the first we take

$$u_i = d_i \cos(kx_3), \quad v_i = d_i \sin(kx_3), \quad (2.8)$$

with  $d_i(t, \rho)$  real while in the second we take

$$u_i = d_i e^{ikx_3}, \quad v_i = id_i e^{ikx_3}, \quad (2.9)$$

again with  $d_i(t, \rho)$  real. These two cases correspond to what we shall call holographic helical  $p_x$  and  $(p_x + ip_y)$ -wave superconductors, respectively, for reasons we discuss in section 2.3 below. In both cases we find that the equation of motion for the two-form associated with (2.1) leads to three equations. Two of these can be used to solve for  $d_1$  and  $d_2$  in terms of  $d_3$ :

$$d_1 = -\frac{1}{m^2 + k^2} \left( \frac{im}{\rho^2} \mathcal{D}_t + k \mathcal{D}_\rho \right) d_3, \quad d_2 = \frac{1}{m^2 + k^2} (ik \mathcal{D}_t - m \rho^2 \mathcal{D}_\rho) d_3, \quad (2.10)$$

and we find that  $d_3$  satisfies the second order equation

$$\left( \mathcal{D}^2 - L^2(m^2 + k^2) + \frac{kq}{3\sqrt{2}m} \right) d_3 = 0. \quad (2.11)$$

In these equations  $\mathcal{D}$  is the unit radius  $AdS_2$  derivative given by

$$\mathcal{D}_\mu \equiv \nabla_\mu + i \frac{q}{\sqrt{3}} A_\mu. \quad (2.12)$$

We thus conclude that these modes have an effective (unit-radius)  $AdS_2$  mass given by

$$M^2 = L^2 (k^2 + m^2) - \frac{kq}{3\sqrt{2}m} - \frac{q^2}{18}. \quad (2.13)$$

The lightest mode is for  $k_{min} = \sqrt{2}q/m$  and has

$$M_{min}^2 = \frac{m^2}{12} - \frac{q^2}{18m^2} (3 + m^2). \quad (2.14)$$

For  $q^2 > 3m^2/2$  we see that this violates the  $AdS_2$  BF bound  $M^2 \geq -1/4$  and the instability will necessarily break the translational invariance of the background solution (2.6). For the case of the Romans' theory with  $m = q = 1$  we have  $M_{min}^2 = -5/36$  which satisfies the BF bound.

## 2.2 Zero modes for the AdS-RN black brane

The instabilities in the  $AdS_2 \times \mathbb{R}^3$  region that we saw in the last subsection will show up as instabilities of the AdS-RN black brane solution (2.4) at some finite temperature depending on  $k$ . More generally, at the temperature at which any instability appears the linearised perturbations about the AdS-RN black hole will, generically, admit a static normalisable zero mode, corresponding to the existence of a new branch of solutions. We now analyse such zero modes.

We again consider a perturbation of the two-form as in (2.7). We also consider the two cases (2.8), (2.9) where the  $d_i$  are real, time independent functions,  $d_i = d_i(r)$ , satisfying appropriate boundary conditions at the black hole event horizon at  $r = r_+$  and at the  $AdS_5$  boundary  $r \rightarrow \infty$ . After substituting into the two-form equation of motion (2.3) we again find that we can solve for  $d_1$  and  $d_2$

$$\begin{aligned} d_1 &= \frac{1}{f(m^2 r^2 + k^2)} \left( \frac{q}{\sqrt{3}} m r a d_3 - k f d_3' \right) \\ d_2 &= -\frac{1}{m^2 r^2 + k^2} \left( \frac{q}{\sqrt{3}} k a d_3 + m r f d_3' \right) \end{aligned} \quad (2.15)$$

and that  $d_3$  satisfies the second order ODE

$$\left( \frac{m r f}{(m^2 r^2 + k^2)} d_3' \right)' + \left( \frac{m r q^2 a^2}{3 f (m^2 r^2 + k^2)} - \frac{2 m^2 r k q a}{\sqrt{3} (m^2 r^2 + k^2)^2} - \frac{m}{r} \right) d_3 = 0. \quad (2.16)$$

One can check that for the spatially homogeneous case, when  $k = 0$ , this is compatible with equation (3.12) of [11].

At the black hole event horizon we impose the boundary conditions

$$d_3 = d_3^0 + \mathcal{O}(r - r_+) \quad (2.17)$$

and we use the linearity of the ODE to choose  $d_3^0 = 1$ . It is straightforward to check that the two-form  $C$  and its field strength  $H$  are both well defined at  $r = r_+$ . As  $r \rightarrow \infty$  we deduce that the asymptotic behaviour is of the form

$$d_3 = w_1 r + \cdots w_2 r^{-1} + \cdots. \quad (2.18)$$

We are only interested in modes in which the operator dual to the two-form  $C$  spontaneously breaks the symmetry and so we demand  $w_1 = 0$ .

We now return to the ODE (2.16) with the boundary conditions (2.17), (2.18), which we have solved using a shooting method. Since we have imposed two boundary conditions on the second order ODE, for a given  $k$  we expect zero mode solutions

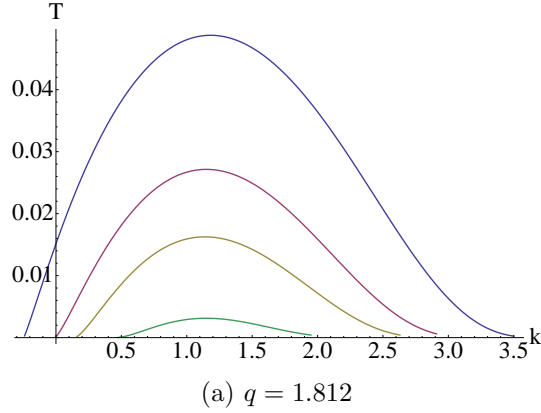


Figure 1: Plots of critical temperatures  $T$  versus  $k$  for the existence of normalisable static perturbations of the two-form about the AdS-RN black brane (2.4) for  $m = 1$  and, from top to bottom,  $q = 2$ ,  $q = 1.812$ ,  $q = 1.7$  and  $q = 1.5$ . We have set  $\mu = \sqrt{3}$ .

to exist, if at all, at a specific temperature. We have made plots in Figure 1 of the temperature  $T$  versus  $k$  for which a normalisable mode exists for a representative range of the two-form charge  $q$  and with  $m = 1$ . The scale has been set by fixing  $\mu = \sqrt{3}$ . The maximum of each curve indicate the critical temperature  $T_c$  at which a spatially modulated zero mode first appears. At  $T = T_c$  a new branch of helical superconducting black branes will appear with the spatial modulation, near  $T = T_c$ , set by  $k_c$ , the value of  $k$  at  $T_c$ .

Observe that, for fixed  $m$ , as we increase  $q$ , we raise  $T_c$ . This mirrors the fact that the corresponding modes in the  $AdS_2 \times \mathbb{R}^3$  region have bigger violations of the BF bound, as we see from (2.14). Similarly fixing  $q$  and increasing  $m$  we find that  $T_c$  decreases again mirroring (2.14). The case  $q = m = 1$  is interesting since these are the values relevant for Romans' theory. While we know that there are no violations of the BF bound in the  $AdS_2 \times \mathbb{R}^3$  region for this case, it is interesting to ask if there are zero modes in the full black hole solution. We have not yet managed to stabilise our numerics for this case, but it is clear from Figure 1 that if they do exist, they will certainly be at very low temperatures.

Another case worth commenting on further is  $m = 1$  and  $q \approx 1.812$ . For this case, we see from Figure 1 that the range of  $k$  extends to include instabilities with  $k = 0$ . Furthermore, for  $m = 1$  it is the smallest value of  $q$  for which this happens. In [11] spatially homogeneous instabilities with  $k = 0$  were considered with  $m = 1$  and variable  $q$  and it was argued that the value  $q \approx 1.8$  is associated with a quantum critical point because for this value a  $k = 0$  mode is just becoming unstable at zero temperature. However, in the larger set of perturbations with  $k \neq 0$  that we are



considering here, we find that for this value of  $q$  modes with  $k > 0$  become unstable at higher temperatures than the  $k = 0$  mode and hence the value  $q \approx 1.8$  is not singled out physically.

### 2.3 The character of the $p$ -wave superconductors

We have established that there will be a new branch of superconducting black holes appearing at  $T = T_c$  corresponding to the linearised perturbation of  $C$  given by (2.7) with  $k = k_c$  for both (2.8) and (2.9). Since  $d_3$  is the only independent degree of freedom, the order parameter operator in the CFT dual to  $C$  is governed by the  $d_3$  dependent terms in (2.7). By taking the Hodge dual of the last term in (2.1) shows that the operator is picking out a direction in the  $x_1, x_2$  plane, corresponding to  $p$ -wave order, that rotates helically as one goes along the  $x_3$  direction. Let us now comment on the difference between the two cases (2.8) and (2.9).

First consider the case (2.8). Recall that when  $k = 0$  this is an ansatz that was already considered in [11] and can be referred to as a holographic  $p_x$ -wave superconductor. In particular, when  $k = 0$ , after taking the Hodge-dual of the last term in (2.1) we see that the operator dual to  $C$  is picking out the (real)  $dx_1$  direction. Furthermore, from (2.1) we can deduce that at second order the perturbation of  $C$  will contribute to the energy momentum tensor,  $T_{\mu\nu}$ , with  $T_{11} \neq T_{22}$  (and  $T_{12} = 0$ ) and hence the isotropy of the metric in the  $x_1, x_2$  plane will be broken, just as in the original holographic  $p_x$ -wave superconductor of [9]. Allowing  $k \neq 0$  in (2.8) it is clear that the dual operator picks out a direction in the  $x_1, x_2$  plane that rotates helically as we go along the  $x_3$  direction. It is thus natural to call these holographic helical  $p_x$ -wave superconductors. It is interesting to point out that when  $k \neq 0$  we have  $T_{12} \neq 0$ , which we will provide an extra challenge in constructing fully back-reacted solutions.

We next consider the case (2.9). When  $k = 0$  the dual operator is now picking out the complex  $dx_1 + idx_2$  direction and when  $k \neq 0$  this is helically winding in the  $x_3$  direction. An important difference with the previous case is that the isotropy of the metric in the  $x_1, x_2$  plane is now preserved since  $T_{11} = T_{22}$  and  $T_{12} = 0$  for all  $k$ . This isotropy is a feature of the original holographic  $(p_x + ip_y)$ -wave superconductors of [8] and so it is natural to call our  $x_3$ -dependent generalisations holographic helical  $(p_x + ip_y)$ -wave superconductors.

### 3 The $SU(2) \times U(1)$ model

We now consider a  $D = 5$  theory coupling a metric, a  $U(1)$  gauge field  $B$  and  $SU(2)$  gauge-fields  $A^\alpha$  with Lagrangian

$$\mathcal{L} = (R + 12) * 1 - \frac{1}{2} * G \wedge G - \frac{1}{2} * F^\alpha \wedge F^\alpha - \frac{\gamma}{2} F^\alpha \wedge F^\alpha \wedge B, \quad (3.1)$$

where the field strengths are

$$G = dB, \quad F^\alpha = dA^\alpha - \frac{g}{\sqrt{2}} \epsilon_{\alpha\beta\gamma} A^\beta \wedge A^\gamma. \quad (3.2)$$

The equations of motion are given by

$$\begin{aligned} R_{\mu\nu} &= -4g_{\mu\nu} + \frac{1}{2} \left( G_\mu{}^\rho G_{\nu\rho} - \frac{1}{6} g_{\mu\nu} G_{\rho\sigma} G^{\rho\sigma} \right) + \frac{1}{2} \left( F_\mu{}^\rho F_{\nu\rho} - \frac{1}{6} g_{\mu\nu} F_{\rho\sigma} F^{\rho\sigma} \right) \\ d * G &= -\frac{\gamma}{2} F^\alpha \wedge F^\alpha \\ D * F^\alpha &= -\gamma F^\alpha \wedge G, \end{aligned} \quad (3.3)$$

where  $D * F^\alpha = d * F^\alpha + \sqrt{2} g \epsilon_{\alpha\beta\gamma} A^\gamma \wedge * F^\beta$ . The equations of motion admit a unit radius  $AdS_5$  solution with vanishing gauge fields. The corresponding dual  $d = 4$  CFT now has  $SU(2) \times U(1)$  global symmetry and  $A^\alpha$  and  $B$  are dual to the corresponding conserved currents, respectively, each with  $\Delta = 3$ . When  $\gamma = g = 1$ , the black hole backgrounds and the *linearised* perturbations that we consider for this model are also relevant for Romans' theory of  $N = 4^+$   $SU(2) \times U(1)$  gauged supergravity, as we discuss in section 4. The general class of models (3.1) was recently studied in [27] in the context of spatially homogenous  $p$ -wave superconductors, where the competition of  $p_x$ -wave and  $(p_x + ip_y)$ -wave superconductivity was explored. We will comment on some of the results of [27] later.

The equations of motion admit the electrically charged  $AdS$  black brane solutions

$$\begin{aligned} ds^2 &= -f(r) dt^2 + \frac{dr^2}{f(r)} + r^2 (dx_1^2 + dx_2^2 + dx_3^2), \\ B &= b(r) = \mu_1 \left( 1 - \frac{r_+^2}{r^2} \right) dt, \quad A^3 = a(r) dt = \mu_2 \left( 1 - \frac{r_+^2}{r^2} \right) dt, \\ f &= r^2 - \frac{r_+^4}{r^2} + \frac{1}{3} (\mu_1^2 + \mu_2^2) \left( \frac{r_+^4}{r^4} - \frac{r_+^2}{r^2} \right). \end{aligned} \quad (3.4)$$

Note that we have allowed chemical potentials,  $\mu_i$ , for two  $U(1)$  factors in the global  $U(1) \times SU(2)$  symmetry group of the dual CFT. Generically, the background (3.4) breaks the global symmetry to  $U(1) \times U(1)$  and the superconducting instabilities that

we consider will break this to a single  $U(1)$ . As we explain in section 4 the solution with  $\mu_2 = \sqrt{2}\mu_1$  can be embedded in string or M-theory via Romans' theory.

The temperature of these black holes is given by  $T = (6r_+^2 - \mu_1^2 - \mu_2^2)/6\pi r_+$  with the extremal limit achieved for  $\sqrt{6}r_+^{ext} = \sqrt{\mu_1^2 + \mu_2^2}$ . The near horizon limit of the zero temperature black hole is given by the  $AdS_2 \times \mathbb{R}^3$  solution

$$\begin{aligned} ds_5^2 &= L^2 \left( -\rho^2 dt^2 + \frac{d\rho^2}{\rho^2} \right) + dx_1^2 + dx_2^2 + dx_3^2, \\ A^3 &= \frac{\sqrt{1-\chi^2}}{\sqrt{6}} r dt, \quad B = \frac{\chi}{\sqrt{6}} \rho dt, \end{aligned} \quad (3.5)$$

with  $L^2 = 1/12$  and  $0 \leq \chi \leq 1$ . Note that to obtain this we have defined  $\chi = \mu_1/\sqrt{\mu_1^2 + \mu_2^2}$  and rescaled  $t \rightarrow \frac{1}{12}t$  and  $x_i \rightarrow x_i/r_+^{ext}$ . We also note that for Romans' theory we should take  $\chi = 1/\sqrt{3}$ . In [27] only black holes with  $\chi = 0$  (i.e.  $\mu_1 = 0$ ) were studied.

### 3.1 Instabilities for $AdS_2 \times \mathbb{R}^3$

Consider the perturbation of the  $SU(2)$  gauge-fields about the  $AdS_2 \times \mathbb{R}^3$  solution (3.5) of the form

$$\delta A^1 + i \delta A^2 = w_1 dx_1 + w_2 dx_2, \quad (3.6)$$

with  $w_i$  being complex functions of  $(t, \rho, x_3)$ . At the linearised level this perturbation decouples from other perturbations. We again consider two types of spatially modulated perturbations.

The first ansatz is

$$w_1 = w \cos(kx_3), \quad w_2 = -w \sin(kx_3), \quad (3.7)$$

with  $w$  a real function of  $x^\mu = (t, \rho)$ . Notice that when  $k = 0$ , we obtain a  $D = 5$  analogue of the ansatz for the  $p_x$ -wave holographic superconductors of [9] (used in [27]), which at higher orders break the isotropy of the metric in the  $x_1, x_2$  plane. Switching on  $k \neq 0$  gives rise to holographic helical  $p_x$ -wave order. The second ansatz is

$$w_1 = w e^{ikx_3}, \quad w_2 = i w e^{ikx_3}, \quad (3.8)$$

with  $w$  a real function of  $x^\mu = (t, \rho)$ . When  $k = 0$ , we obtain a  $D = 5$  analogue of the ansatz for the  $(p_x + ip_y)$ -wave holographic superconductors of [8] (used in [27]), which preserves a diagonal subgroup of the rotations in the  $x_1, x_2$  plane and a  $U(1) \subset SU(2)$ .

The corresponding metric preserves the isotropy of the metric in the  $x_1, x_2$  plane. Switching on  $k \neq 0$  gives rise to holographic helical  $(p_x + ip_y)$ -wave order.

After substituting into the equations of motion for the gauge-fields, (3.3), we find that in both cases, at linearised order,  $w$  satisfies

$$\left(\mathcal{D}^2 - L^2 k^2 - \frac{1}{\sqrt{6}} \chi \gamma k\right) w = 0, \quad (3.9)$$

where we are using the covariant derivative on the unit-radius  $AdS_2$  space given by

$$\mathcal{D}_\mu \equiv \nabla_\mu - ig\sqrt{2}A_\mu^3. \quad (3.10)$$

We thus conclude that these modes have an effective (unit radius)  $AdS_2$  mass given by

$$M^2 = L^2 k^2 + \frac{g^2}{3} (\chi^2 - 1) - \frac{1}{\sqrt{6}} \chi k \gamma. \quad (3.11)$$

The minimum mass occurs at  $k_{min} = \sqrt{6} \chi \gamma$  giving

$$M_{min}^2 = \frac{g^2}{3} (\chi^2 - 1) - \frac{1}{2} \chi^2 \gamma^2. \quad (3.12)$$

By suitable choice of backgrounds specified by  $\chi$  and theories specified by  $g, \gamma$  it is easy to violate the BF bound  $M^2 \geq -1/4$ . In Romans' theory, for example, with  $\gamma = g = 1$  and  $\chi = 1/\sqrt{3}$  we have  $M_{min}^2 = -7/18$  which violates the BF bound  $M^2 \geq -1/4$ .

We next consider  $\chi = 1$ . This case has  $\mu_2 = 0$  and hence the background solution is only carrying electric charge with respect to the  $U(1)$  factor and has vanishing  $SU(2)$  fields. Nevertheless, provided that  $\gamma > 1/\sqrt{2}$  we can still violate the BF bound with spatially modulated instabilities. For this case the background preserves the  $U(1) \times SU(2)$  global symmetry and the perturbation preserves the  $U(1)$  but breaks the  $SU(2)$ .

Another interesting case is  $\chi = 0$  ( $\mu_1 = 0$ ) in which the background is only electrically charged with respect to  $U(1) \subset SU(2)$ . In this case the BF bound is violated provided that  $g > \sqrt{3}/2$  and notice that these superconducting modes have  $k_{min} = 0$  and are not spatially modulated. In fact these modes were studied in [27] and we will return to them in section 3.3.

### 3.2 Zero modes for the AdS-RN black brane

We now consider analogous perturbations of the one-forms about the black brane solutions given in (3.4). We again consider the spatially modulated superconducting

perturbations as in (3.6),(3.7),(3.8) but now with  $w$  a time independent, real function of  $r$ . Substituting this ansatz into the  $SU(2)$  gauge field equation of motion in (3.3) leads to, at linearised order, the second order ODE

$$(rfw')' - \left( \frac{k^2}{r} - \frac{2g^2a^2r}{f} - \gamma k b' \right) w = 0. \quad (3.13)$$

We now wish to solve this equation by imposing suitable boundary conditions. At the black hole event horizon,  $r = r_+$ , we have the expansion

$$w = w^0 + \mathcal{O}(r - r_+). \quad (3.14)$$

Observe that the gauge-field is regular when  $w^0 \neq 0$ . Furthermore, since the equation is linear we can choose  $w^0 = 1$ . At  $r \rightarrow \infty$ , the asymptotic  $AdS_5$  boundary, we have the expansion

$$w = g_0 + \dots + g_1 r^{-2} + \dots. \quad (3.15)$$

Since we are interested only in the case in which the  $SU(2)$  global symmetry current in the boundary CFT, dual to  $A^\alpha$ , spontaneously develops an expectation value, we demand that  $g_0 = 0$ .

We have numerically solved equation (3.13) with these boundary conditions using a shooting method. The results are shown in Figure 2 for the Romans' case ( $\gamma = g = 1$ ,  $\mu_2 = \sqrt{2}\mu_1$  and we have scaled  $\mu_1 = 1$ ) where we have plotted the value of the temperature  $T$  for which we can find such a function for any given momentum  $k$ . At  $k = 0$  the value of the temperature is very small, but non-zero. Observe that the highest critical temperature,  $T_c \sim 0.011$  occurs for  $k_c \sim 0.72$ . At this temperature a new branch of black brane solutions will exist that are dual to spatially modulated helical superfluid phases in the boundary CFT.

### 3.3 Spatially modulated neutral instabilities

Before concluding this section we would like to point out that in addition to the superconducting instabilities that we have just discussed, the model (3.1) also has neutral spatially modulated instabilities. That this is the case can quickly be established as follows. Observe that we can consistently truncate  $A^1 = A^2 = 0$  and also set  $A^3 = \sqrt{2}B$  in the equations of motion for (3.1). After rescaling  $\sqrt{3}B = A$ , so that  $A$  has a canonical kinetic term, we find that the truncated model is exactly the same as that studied in [20] (after identifying  $\gamma = 2\sqrt{3}\alpha_{there}$ ). In particular, for large enough values of  $\gamma$  there will be spatially modulated neutral instabilities. In appendix A

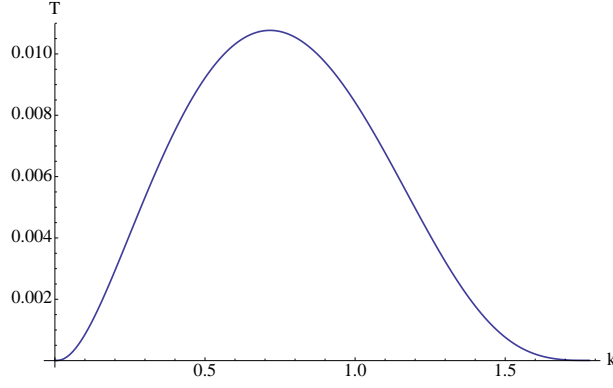


Figure 2: Plot of critical temperatures  $T$  versus  $k$  for the existence of normalisable static perturbations of  $A^1$ ,  $A^2$  about the electrically charged black holes (3.4). The plot is for Romans' theory with  $\gamma = g = 1$  and also  $\mu_2 = \sqrt{2}$  and  $\mu_1 = 1$ .

we have presented a few details of the analysis of these types of instabilities in the  $AdS_2 \times \mathbb{R}^3$  solution (3.4), for general values of  $\chi$ .

The case of  $\chi = 0$  is relevant for the discussion of [27]. In that paper an analysis of the competition between  $p_x$ -wave and  $(p_x + ip_y)$ -wave superconductors was investigated. As we already pointed out above, these superconductors are spatially homogeneous. (Note that to compare one should identify our  $g, \gamma$  with their  $\alpha, \gamma$ .) Our analysis in appendix A shows that when  $\chi = 0$  and  $\gamma > 1$  the black holes will also have neutral spatially inhomogeneous instabilities. It would be interesting to use these results to extend the analysis of [27] to determine what are the true thermodynamically preferred black holes.

## 4 Romans' Theory

Romans'  $\mathcal{N} = 4^+$   $SU(2) \times U(1)$  gauged supergravity [28] has bosonic fields consisting of a metric, gauge fields  $A^\alpha$  and  $B$ , a complex two form  $C$  and a neutral scalar field,  $X$ . Following the conventions of [32] (and setting  $m = 1$ ), the Lagrangian is

$$\begin{aligned} \mathcal{L} = & R * 1 - 3X^{-2} * dX \wedge dX - \frac{1}{2}X^4 * G \wedge G - \frac{1}{2}X^{-2} (*F^\alpha \wedge F^\alpha + *C \wedge \bar{C}) \\ & - \frac{i}{2}C \wedge \bar{H} - \frac{1}{2}F^\alpha \wedge F^\alpha \wedge B + 4(X^2 + 2X^{-1}) * 1, \end{aligned} \quad (4.1)$$

where the field strengths are

$$G = dB, \quad F^\alpha = dA^\alpha - \frac{1}{\sqrt{2}}\epsilon_{\alpha\beta\gamma} A^\beta \wedge A^\gamma, \quad H = dC + iB \wedge C. \quad (4.2)$$

Any solution of the equations of motion, which are explicitly given in [32], can be uplifted on an  $S^5$  to obtain a solution of type IIB supergravity [29]. They can also be uplifted on the general class of  $M_6$  [30] (see also [31]), corresponding to  $N = 2$   $d = 4$  SCFTs, to obtain an infinite class of solutions of  $D = 11$  supergravity [32].

As pointed out in [32] there exists a consistent truncation to minimal five dimensional gauged supergravity after setting  $X = 1$ ,  $C = A^1 = A^2 = 0$  and  $A^3 = \sqrt{2}B$ . Notice that the canonically normalised gauge field for minimal gauged supergravity is then defined as  $A = \sqrt{3}B$ . In particular, we notice that the electrically charged AdS-RN black brane solution (2.4),(2.5) is a solution of Romans' theory (with  $A^3 = \sqrt{2/3}a(r)dt$ ). Notice also that setting  $q = 1$  in (2.2) agrees with (4.2). Similarly setting  $\mu_2 = \sqrt{2}\mu_1$  in (3.4) we also obtain the same AdS-RN black hole solution of Romans' theory after identifying  $\mu = \sqrt{3}\mu_1$ .

We can now consider perturbations about these AdS-RN black brane solutions. We find that the two-form perturbations and the one-form perturbations that we studied in the previous two sections remain decoupled from other perturbations at *linearised* order also within Romans' theory<sup>3</sup>. For the two-form perturbations we commented that, after setting  $q = m = 1$ , if there are spatially modulated zero modes associated with the helical superconductors they would appear at very low temperatures. On the other hand, for the one-form perturbations we saw that, after setting  $\alpha = g = 1$ , the zero modes appear at  $T_c \sim 0.011$ . Thus we conclude that helical superconducting black hole solutions can be found in Romans's theory and hence in string/M-theory. In the next subsection we will show that Romans' theory has yet another instability, of a type first discussed by Gubser and Mitra [33,34], that happens at an even higher temperature than the helical superconducting instability.

As first emphasised in [35], identifying the highest critical temperature at which an instability sets in within a consistent truncation is certainly not sufficient to deduce the thermodynamically preferred phases of the dual CFTs. In principle one needs to construct all of the back reacted brane geometries and calculate their free energies. These include the non-linear branches of black hole solutions associated with the zero modes that we have found, but also possible branchings of these solutions. As in [35] it is possible that the Gubser-Mitra type branch, for example, sprouts a superconducting branch at lower temperatures. Furthermore, there could also be relevant black holes associated with higher KK modes that are outside of the truncation to Romans theory.

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<sup>3</sup>Going beyond linearised order for the two-form helical superconducting instability of section 2, one finds that the scalar field  $X$  is sourced, but not the  $SU(2)$ -gauge fields. For the instability of section 3, the scalar field is sourced, but not the charged two-form.

We know for sure that at temperatures below the Gubser-Mitra critical temperature the dual CFTs cannot be described by the AdS-RN black brane solutions (because they are unstable). This does *not* imply that black hole solutions associated with the superconducting zero modes for the AdS-RN black holes that we have constructed in section 3.2 are physically irrelevant. While it does show these black holes do not appear as thermodynamically preferred states arising from a second order phase transition, it is still possible that they appear after a first order phase transition. It would be interesting to know if this actually happens.

Before discussing the Gubser-Mitra instability we note that Romans' theory admits a one parameter family of electrically charged  $AdS_2 \times \mathbb{R}^3$  solutions with constant scalar field  $X$ , but unlike above,  $X \neq 1$  [28]. It is plausible that these arise as the near horizon limits of zero temperature black brane solutions which have not yet been constructed. In appendix B we show that the one-form instabilities of section 3.2 are present for all of these solutions.

## 4.1 The Gubser-Mitra instability

We now briefly discuss the zero modes associated with the Gubser-Mitra type instability of the AdS-RN black holes within Romans' theory. We consider the following perturbation around the black hole solution (3.4) with  $\mu_2 = \sqrt{2}\mu_1$ :

$$X = 1 + \frac{1}{\sqrt{6}} \delta\phi, \quad A^3 = \sqrt{2}b(r) dt - \frac{1}{\sqrt{3}} \delta b dt, \quad B = b(r) dt + \sqrt{\frac{2}{3}} \delta b dt, \quad (4.3)$$

where  $\delta\phi$  and  $\delta b$  are functions of  $r$ . The equations of motion at linearised order lead to the coupled differential equations

$$\begin{aligned} \frac{1}{r^3} (r^3 f \delta\phi')' + 2b'^2 \delta\phi + 4\delta\phi + 2b' \delta b' &= 0, \\ (2r^3 b' \delta\phi + r^3 \delta b')' &= 0. \end{aligned} \quad (4.4)$$

Near the horizon we impose the regular expansion

$$\delta\phi = \phi_+^{(0)} + \mathcal{O}(r - r_+), \quad \delta b = b_+^{(1)} (r - r_+) + \mathcal{O}(r - r_+)^2 \quad (4.5)$$

and we use the scaling symmetry of the linearised problem to set  $\phi_+^{(0)} = 1$ . The asymptotic behaviour as  $r \rightarrow \infty$  is given by

$$\delta\phi = \bar{v}_1 \log(r) r^{-2} + \cdots + v_1 r^{-2} + \cdots, \quad \delta b = g_0 + \cdots + g_1 r^{-2} + \cdots. \quad (4.6)$$

We are interested in the system spontaneously acquiring expectation values and so we demand that  $\bar{v}_1 = g_0 = 0$ . Using a shooting method, after setting  $\mu_1 = 1$ , we find that the temperature for which such a solution to (4.4) exists is at  $T \approx 1/(2\pi) \approx 0.159$ .



## 5 Final Comments

We have constructed linearised zero modes associated with spatially modulated black holes that are dual to helical  $p$ -wave superconductors. We saw that the equations governing the  $p_x$ -wave and the  $(p_x + ip_y)$ -wave cases were exactly the same at the linearised level. However, these cases will differ at higher orders and it will be interesting to see how they compete, generalising the investigations of [9, 10, 27]. Extending the perturbative analysis beyond the linearised level will also illuminate additional features of the helical superconductors, for example whether or not charge density waves are realised. The details will depend on precisely which model one is considering; in Romans' theory, for example, the scalar field  $X$  will become activated. Of course, it would be most interesting to go beyond a perturbative analysis and construct fully back reacted solutions.

More generally, this work combined with [20, 24, 26] indicates that string/M-theory admits a very rich landscape of spatially modulated electrically and magnetically charged black hole solutions. This leads one to speculate that the generic ground states are not spatially homogeneous as hitherto thought.

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## A Neutral instabilities for the $SU(2) \times U(1)$ model

For the model with  $SU(2) \times U(1)$  symmetry, with Lagrangian given in (3.1), we consider the time independent perturbation

$$\begin{aligned}
 \delta A^3 &= \frac{u(r)}{\sqrt{6}} [\cos(kx_3) dx_1 - \sin(kx_3) dx_2] \\
 \delta B &= \frac{v(r)}{\sqrt{6}} [\cos(kx_3) dx_1 - \sin(kx_3) dx_2] \\
 \delta g_{tx_1} &= r h(r) \cos(kx_3) \\
 \delta g_{tx_2} &= -r h(r) \sin(kx_3)
 \end{aligned} \tag{A.1}$$

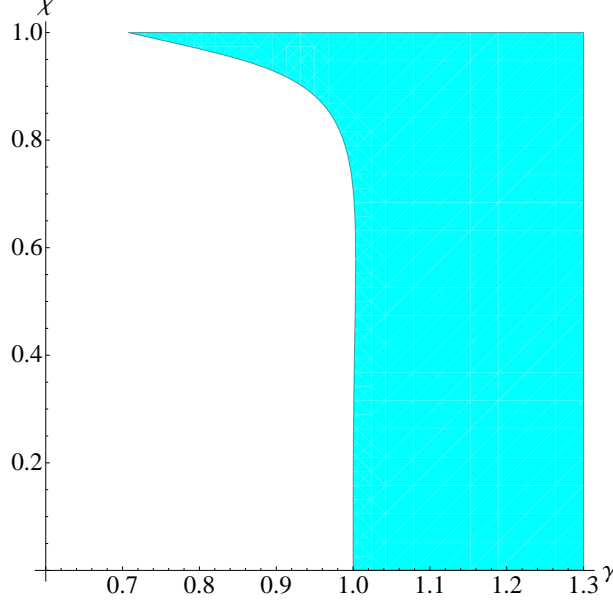


Figure 3: The shaded region indicates values of  $(\chi, \gamma)$  for which the  $AdS_2 \times \mathbb{R}^3$  solutions (3.5) have neutral instabilities of the form (A.1) which break translational and rotational invariance.

around the one parameter family of  $AdS_2 \times \mathbb{R}^3$  solutions given in (3.5). Such perturbations correspond to spatially modulated *non-superconducting* instabilities generalising those studied in [20]. At linearised order, the equations of motion give the coupled linear system of equations

$$\begin{aligned}
 12 (r^2 h')' - k^2 h + 2r (\chi b' + \sqrt{1 - \chi^2} u') &= 0 \\
 12 (r^2 v')' - k^2 v + 144 \chi (rh)' + 2\sqrt{6}k\gamma \sqrt{1 - \chi^2} u &= 0 \\
 12 (r^2 u')' - k^2 u + 144 \sqrt{1 - \chi^2} (rh)' + 2\sqrt{6}k\gamma \sqrt{1 - \chi^2} v + 2\sqrt{6}k\gamma \chi u &= 0 \quad (A.2)
 \end{aligned}$$

To determine the scaling dimensions of the dual operators we look for solutions of the form  $(h, a, b) = \mathbf{v} r^\lambda$ , with  $\mathbf{v}$  a constant vector. Plugging this ansatz in the linear system (A.2) we obtain a matrix equation of the form  $\mathbf{M}\mathbf{v} = 0$ , where  $\mathbf{M}$  is a three by three matrix depending on  $\lambda$ ,  $\chi$ ,  $\gamma$  and  $k$ . The possible values of  $\lambda$  are then determined by the requirement that  $\det \mathbf{M} = 0$  has non-trivial solutions. If any  $\lambda$  has a non-zero imaginary part then the  $AdS_2 \times \mathbb{R}^3$  background is unstable. In Figure 3 we have plotted a shaded region in the  $\chi - \gamma$  plane where these neutral instabilities exist. In all cases, the instability is spatially modulated with  $k \neq 0$ . Note that the  $\chi = 0$  case is relevant for the analysis of [27].

## B Instabilities of a line of $AdS_2 \times \mathbb{R}^3$ solutions

Romans' theory, with Lagrangian (4.1), is known [28] to admit the following one parameter family of  $AdS_2 \times \mathbb{R}^3$  solutions, with constant scalar field,  $X = X_0 > 0$ :

$$ds_5^2 = L^2 ds^2 \left( -\rho^2 dt^2 + \frac{d\rho^2}{\rho^2} \right) + dx_1^2 + dx_2^2 + dx_3^2$$

$$A^3 = X_0^{3/2} \frac{\sqrt{1+X_0^3}}{\sqrt{2}(2+X_0^3)} \rho dt, \quad B = \frac{1}{\sqrt{2}X_0^{3/2}(2+X_0^3)} \rho dt \quad (\text{B.1})$$

where

$$L^2 = \frac{X_0}{4(2+X_0^3)} \quad (\text{B.2})$$

Notice that when  $X_0 = 1$  we recover the solution given in (3.5) after setting  $\mu_2 = \sqrt{2}\mu_1$ . We now show that the one-form perturbations of the type discussed in section 3.1 violate the  $AdS_2$  BF bound. On the other hand, the two-form perturbations of the type discussed in section 2.1 do not lead to such a violation for any value of  $X_0$ .

We again consider the one-form fluctuations as in (3.6),(3.7),(3.8) with  $w$  a real function of  $t, \rho$ . Following a similar analysis as in section (3.5) we find effective (unit-radius)  $AdS_2$  masses given by

$$M^2 = k^2 + X_0^2 \left( -4 + \frac{4}{2+X_0^3} \right) - \frac{2\sqrt{2}}{\sqrt{X_0}} k \quad (\text{B.3})$$

Notice that this always develops a minimum at

$$k_{min} = \sqrt{2X_0^{-1}} \quad (\text{B.4})$$

with

$$M_{min}^2 = -\frac{2+3X_0^3+2X_0^6}{2(2+X_0^3)^2} < -\frac{1}{4} \quad (\text{B.5})$$

which is always violating the BF bound. It is worth noticing that this is a monotonically decreasing function of  $X_0$  asymptoting to  $-1$ .

## References

- [1] S. S. Gubser, “Breaking an Abelian Gauge Symmetry Near a Black Hole Horizon,” *Phys. Rev.* **D78** (2008) 065034, [arXiv:0801.2977 \[hep-th\]](#).
- [2] S. A. Hartnoll, C. P. Herzog, and G. T. Horowitz, “Building a Holographic Superconductor,” *Phys. Rev. Lett.* **101** (2008) 031601, [arXiv:0803.3295 \[hep-th\]](#).

- [3] S. A. Hartnoll, C. P. Herzog, and G. T. Horowitz, “Holographic Superconductors,” *JHEP* **12** (2008) 015, [arXiv:0810.1563 \[hep-th\]](#).
- [4] F. Denef and S. A. Hartnoll, “Landscape of superconducting membranes,” *Phys. Rev.* **D79** (2009) 126008, [arXiv:0901.1160 \[hep-th\]](#).
- [5] J. P. Gauntlett, J. Sonner, and T. Wiseman, “Holographic superconductivity in M-Theory,” *Phys. Rev. Lett.* **103** (2009) 151601, [arXiv:0907.3796 \[hep-th\]](#).
- [6] J. P. Gauntlett, J. Sonner, and T. Wiseman, “Quantum Criticality and Holographic Superconductors in M- theory,” *JHEP* **02** (2010) 060, [arXiv:0912.0512 \[hep-th\]](#).
- [7] S. S. Gubser, C. P. Herzog, S. S. Pufu, and T. Tesileanu, “Superconductors from Superstrings,” *Phys. Rev. Lett.* **103** (2009) 141601, [arXiv:0907.3510 \[hep-th\]](#).
- [8] S. S. Gubser, “Colorful horizons with charge in anti-de Sitter space,” *Phys. Rev. Lett.* **101** (2008) 191601, [arXiv:0803.3483 \[hep-th\]](#).
- [9] S. S. Gubser and S. S. Pufu, “The gravity dual of a p-wave superconductor,” *JHEP* **11** (2008) 033, [arXiv:0805.2960 \[hep-th\]](#).
- [10] M. M. Roberts and S. A. Hartnoll, “Pseudogap and time reversal breaking in a holographic superconductor,” *JHEP* **08** (2008) 035, [arXiv:0805.3898 \[hep-th\]](#).
- [11] F. Aprile, D. Rodriguez-Gómez, and J. G. Russo, “P-Wave Holographic Superconductors and Five-Dimensional Gauged Supergravity,” *JHEP* **01** (2011) 056, [arXiv:1011.2172 \[hep-th\]](#).
- [12] M. Ammon, J. Erdmenger, M. Kaminski, and P. Kerner, “Superconductivity from gauge/gravity duality with flavor,” *Phys. Lett.* **B680** (2009) 516–520, [arXiv:0810.2316 \[hep-th\]](#).
- [13] P. Basu, J. He, A. Mukherjee, and H.-H. Shieh, “Superconductivity from D3/D7: Holographic Pion Superfluid,” *JHEP* **11** (2009) 070, [arXiv:0810.3970 \[hep-th\]](#).
- [14] K. Peeters, J. Powell, and M. Zamaklar, “Exploring colourful holographic superconductors,” *JHEP* **09** (2009) 101, [arXiv:0907.1508 \[hep-th\]](#).

- [15] A. P. Mackenzie and Y. Maeno, “The superconductivity of  $\text{Sr}_2\text{RuO}_4$  and the physics of spin-triplet pairing,” *Rev. Mod. Phys.* **75** (May, 2003) 657–712. <http://link.aps.org/doi/10.1103/RevModPhys.75.657>.
- [16] Y. Shin, C. H. Schunck, A. Schirotzek, and W. Ketterle, “Phase diagram of a two-component Fermi gas with resonant interactions,” *Nature* **451** (2008) 689.
- [17] F. Benini, C. P. Herzog, R. Rahman, and A. Yarom, “Gauge gravity duality for d-wave superconductors: prospects and challenges,” *JHEP* **11** (2010) 137, [arXiv:1007.1981 \[hep-th\]](#).
- [18] P. Fulde and R. A. Ferrell, “Superconductivity in a Strong Spin-Exchange Field,” *Phys. Rev.* **135** (1964) A550–563.
- [19] A. I. Larkin and Y. N. Ovchinnikov, “Nonuniform state of superconductors,” *Zh. Eksp. Teor. Fiz.* **47** (1964) 1136–1146.
- [20] S. Nakamura, H. Ooguri, and C.-S. Park, “Gravity Dual of Spatially Modulated Phase,” *Phys. Rev.* **D81** (2010) 044018, [arXiv:0911.0679 \[hep-th\]](#).
- [21] H. Ooguri and C.-S. Park, “Holographic End-Point of Spatially Modulated Phase Transition,” *Phys. Rev.* **D82** (2010) 126001, [arXiv:1007.3737 \[hep-th\]](#).
- [22] H. Ooguri and C.-S. Park, “Spatially Modulated Phase in Holographic Quark-Gluon Plasma,” *Phys. Rev. Lett.* **106** (2011) 061601, [arXiv:1011.4144 \[hep-th\]](#).
- [23] S. K. Domokos and J. A. Harvey, “Baryon number-induced Chern-Simons couplings of vector and axial-vector mesons in holographic QCD,” *Phys. Rev. Lett.* **99** (2007) 141602, [arXiv:0704.1604 \[hep-ph\]](#).
- [24] A. Donos and J. P. Gauntlett, “Holographic striped phases,” *JHEP* **08** (2011) 140, [arXiv:1106.2004 \[hep-th\]](#).
- [25] O. Bergman, N. Jokela, G. Lifschytz, and M. Lippert, “Striped instability of a holographic Fermi-like liquid,” [arXiv:1106.3883 \[hep-th\]](#).
- [26] A. Donos, J. P. Gauntlett, and C. Pantelidou, “Spatially modulated instabilities of magnetic black branes,” [arXiv:1109.0471 \[hep-th\]](#).

- [27] L. A. P. Zayas and D. Reichmann, “A Holographic Chiral  $p_x + ip_y$  Superconductor,” [arXiv:1108.4022 \[hep-th\]](#).
- [28] L. J. Romans, “Gauged N=4 supergravities in five-dimensions and their magnetovac backgrounds,” *Nucl. Phys.* **B267** (1986) 433.
- [29] H. Lu, C. N. Pope, and T. A. Tran, “Five-dimensional N = 4, SU(2) x U(1) gauged supergravity from type IIB,” *Phys. Lett.* **B475** (2000) 261–268, [arXiv:hep-th/9909203](#).
- [30] H. Lin, O. Lunin, and J. M. Maldacena, “Bubbling AdS space and 1/2 BPS geometries,” *JHEP* **10** (2004) 025, [arXiv:hep-th/0409174](#).
- [31] E. O Colgain, J.-B. Wu, and H. Yavartanoo, “On the generality of the LLM geometries in M-theory,” *JHEP* **04** (2011) 002, [arXiv:1010.5982 \[hep-th\]](#).
- [32] J. P. Gauntlett and O. Varela, “D=5  $SU(2) \times U(1)$  Gauged Supergravity from D=11 Supergravity,” *JHEP* **02** (2008) 083, [arXiv:0712.3560 \[hep-th\]](#).
- [33] S. S. Gubser and I. Mitra, “Instability of charged black holes in anti-de Sitter space,” [arXiv:hep-th/0009126](#).
- [34] S. S. Gubser and I. Mitra, “The evolution of unstable black holes in anti-de Sitter space,” *JHEP* **08** (2001) 018, [arXiv:hep-th/0011127](#).
- [35] A. Donos and J. P. Gauntlett, “Superfluid black branes in  $AdS_4 \times S^7$ ,” *JHEP* **06** (2011) 053, [arXiv:1104.4478 \[hep-th\]](#).